Phys 410
Fall 2013
Lecture #18 Summary
31 October, 2013

The only quantity not controlled in a typical scattering experiment is the impact parameter b of the projectile with respect to the target particle. The impact parameter is the distance of closest approach to the target particle, assuming no forces of interaction cause the projectile to change from its initial direction. Because we cannot control the impact parameter, we have to perform many experiments in which all possible values of b are employed for the incident beam of projectiles. We then give a statistical description of the resulting scattering. With such a description, we can write the number of particles scattered N_{scatt} in terms of the number of particles incident N_{inc} as $N_{scatt} = N_{inc} n_{target} \sigma$, where n_{target} is the density of target particles projected into the two-dimensional plane ($n_{target} \sim 1/m^2$) and σ is defined as the scattering cross section of each particle. σ is often measured in units of 'barns', which is $10^{-28} m^2$. We can generalize the concept of cross section to any process, including capture ($\sigma_{capture}$), ionization ($\sigma_{ionization}$), fission ($\sigma_{fission}$), etc. This is done by using the definition $N_{scatt,x} = N_{inc} n_{target} \sigma_x$ for process "x".

Experiments start with a beam of projectile particles of identical structure and equal initial momenta and energy. The projectiles enter the target with all possible values of impact parameter. One then measures how many particles come out with angle of exit θ, φ and also the energy and momentum of the exiting particle. Our job is to identify the force of interaction between the projectile and target particles from the number of particles scattered through angle θ, φ, for all possible angles. We write the 'angle-resolved' scattering cross section as $N_{scatt}(into\ d\Omega\ around\ \theta, \varphi) = N_{inc} n_{target} \frac{d\sigma}{d\Omega}(\theta, \varphi) d\Omega$, where $\frac{d\sigma}{d\Omega}(\theta, \varphi)$ is called the differential scattering cross section. Note that the element of differential solid angle is $d\Omega = 2\pi \sin\theta\, d\theta d\varphi$. We expect that if this quantity is integrated over all possible exiting angles, we should recover the total scattering cross section for this process: $\sigma = \iint \frac{d\sigma}{d\Omega}(\theta, \varphi)\, d\Omega$. We shall assume that all scattering potentials are spherically symmetric, hence there will be no dependence on the φ coordinate.

To find $\frac{d\sigma}{d\Omega}(\theta, \varphi)$ we compare the area covered by the incident particles at impact parameters between b and $b + db$ (i.e. $d\sigma = 2\pi b db$) to the solid angle subtended by the exiting beam of particles (i.e. $d\Omega = 2\pi \sin\theta\, d\theta$) to arrive at $\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$. To find this, we need to calculate the trajectory of a projectile particle for every possible impact parameter. We then did the example of a point particle elastically scattering from a fixed hard sphere of radius R and found that $\frac{d\sigma}{d\Omega} = \frac{R^2}{4}$, which is independent of angle! The total scattering cross section is just $\sigma = \pi R^2$, which is just the cross-sectional area of the sphere.

We considered Rutherford scattering and calculated the differential scattering cross section for scattering of an alpha particle (charge q) from a Au nucleus (charge Q). The two particles interact by means of the Coulomb force, which is parameterized as $F = \gamma/r^2$, with $\gamma = qQ/4\pi\varepsilon_0$. The orbit is a hyperbola, characteristic of an orbit of two particles interacting by means of a central inverse-square-law force with energy $E > 0$. By calculating the change in momentum of the alpha particle $|\Delta\vec{p}|$ in two ways, we found the relationship between the impact parameter and the scattering angle: $b = \frac{\gamma}{mv^2}\cot\theta/2$, where m is the alpha particle mass and v is its initial speed of the alpha when far from the nucleus. Putting this into the formula for the differential scattering cross section, $\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$, we find: $\frac{d\sigma}{d\Omega} = \left(\frac{qQ/4\pi\varepsilon_0}{4E\sin^2(\theta/2)}\right)^2$.

This result was tested experimentally by Geiger and Marsden, who showed that the scattering rate scaled with n_{target} (by varying the thickness of the foil), scaled as $1/E^2$ (by varying the energy of the incident alpha particles) , scaled as $\frac{1}{\sin^4(\theta/2)}$ (by measuring the number of particles scattered vs. outgoing angle), and scaled as Z^2, where Q=+Ze is the nuclear charge.

Note that because $\frac{d\sigma}{d\Omega} \sim q^2Q^2$, the scattered particle distribution is insensitive to whether the Coulomb interaction is attractive or repulsive. Also, the agreement for the angular dependence of $\frac{d\sigma}{d\Omega}$ with data suggests that the Coulomb force has the simple $1/r^2$ dependence even down to nuclear length scales. Finally, the total scattering cross section calculated from this $\frac{d\sigma}{d\Omega}$ diverges. This is because the bare Coulomb force is infinitely long ranged. In reality, the Coulomb force of the nucleus is screened out by the electron cloud of the atom, on the length scale of one nm, or less. When this screening is taken into account the total scattering cross section becomes finite, as observed.

These calculations assume that the alpha particle only undergoes one scattering event in the material (the Born scattering approximation). In addition, because of the electron screening, when an alpha particle is near one nucleus, it is insensitive to all the other nuclei because they are ‘cloaked’ by their neutralizing electron clouds.